QSES's and the Quantum Jump

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The stochastic methods in Hilbert space have been used both from a fundamental and a practical point of view. The result reported here concerns the application of these methods to model the evolution of quantum systems and does not enter into the question of their fundamental or practical status. It can easily be stated as follows: Once a quantum stochastic evolution scheme is assumed, the incompatibility of the Markov property with the notion of quantum jumps is easily established.

Key words: Stochastic Evolution; Quantum Jump; Mackovian.

The paper [1] is devoted to mathematically prove the result of the present paper, for which we put forward the necessary definitions and theorems.

The mathematical framework is the well-known Hilbert-space-valued stochastic processes formalism [2, 3], which hereafter we will call Quantum Stochastic Evolution Schemes (QSES's). The physical idea behind this assumption is rather clear: a quantum system (thus described by a vector belonging to a Hilbert space) evolves in a nondeterministic fashion, i.e. only the probability of evolution towards different states can be calculated.

The key tool is the transition probability in a time t from an initial state ϕ to a final state ψ, in which time homogeneity is implicitly assumed:

$$P(t;\,\phi,\,\psi) \equiv P(0,\,\phi,\,t,\,\psi) = P(u,\,\phi,\,t+u,\,\psi). \quad (1)$$

With this tool, the Markov property is straightforwardly stated as*

$$\int_{\mathcal{H}} P(t;\phi,\psi)\,P(s;\psi,\varphi)\,\mu(\mathrm{d}\psi) = P(t+s;\phi,\varphi)$$
$$\forall \phi,\varphi \in \mathcal{H},\ \forall t,\,s \in \mathbb{R}^{+}. \quad (2)$$

The necessary result is the following translation to the QSES language of a well-known theorem in time-continuous Markov chain theory [4]:

Let $P(\cdot;\,\varphi,\,\psi)$ be a stochastic transition matrix corresponding to a markovian QSES. Then $P(\cdot;\,\varphi,\,\psi)$ is continuous in $(0,\infty)$ for all $\varphi,\,\psi \in \mathfrak{H}$ if and only if the following limit exists:

$$\lim_{t\to 0^+} P(t;\phi,\psi) = g(\phi,\psi). \quad (3)$$

The utility of this result on the possibility of checking the continuity of a stochastic matrix at every point through the study of a simple limit at the origin. The only assumed hypothesis is the Markov property.

The main physical hypothesis we are analysing is the quantum jump, one of the most controversial aspects of quantum theory. Using the mathematical language assumed, we state that a QSES reflects the quantum jump if its transition probability satisfies

$$P(t;\phi,\psi) = \begin{cases} \begin{cases} 1 & \text{if } 0 \le t \le t' \text{ and } \psi = \hat{U}(t)\,\phi, \\ 0 & \text{if } 0 \le t \le t' \text{ and } \psi \ne \hat{U}(t)\,\phi, \end{cases} \\ h(\phi,\psi)(\ne 0) \text{ if } t > t' \text{ and it is possible} \\ \quad \text{that } \|\psi - \phi\| > \varepsilon \text{ for some } \varepsilon > 0\,, \end{cases} \quad (4)$$

where $\hat{U}(t)$ is the usual quantum-mechanical evolution operator.

Notice that $h(\varphi,\,\psi) = |(\varphi,\,\psi)|^2$ should be expected in order to reproduce the reduction postulate. An excellent framework for understanding the quantum jump is P. Mittelstaedt's analysis of the measurement process [5], which clearly displays the discontinuity of the state collapse at the instant of objectification and reading of the

* From a strict mathematical point of view this is not the Markov property, but an immediate consequence of it, namely the Chapman-Kolmogorov equation. We do not care about these refinements.

value of an observable, a feature we are trying to confront with the Markovianity within the QSES's framework.

The theorem (3) provides a very adequate frame to confront the Markovian QSES and the quantum jump hypothesis. We in no case adopt a priori attitudes about the nature of the quantum collapse, but just try to enlighten its possible compatibility with the mathematical nature assumed to represent quantum systems.

Once we keep in mind the previous mathematical result, it is easy to prove the following theorem:

> *Let S be a quantum system described by a time-homogeneous QSES. If S is subjected to quantum jumps, then its QSES is non-Markovian.*

It is straightforward to convince oneself that the central idea of the proof is the existence of the limit at the origin of time in order to apply the mentioned theorem. Though apparently trivial, the question of the existence of such a limit, which in the general theory of Markov chains is called the *standard condition*, deserves some careful attention which is to be paid in the following section.

Physically the notion of standardization is rather clear: the state of a quantum system does not change if time has hardly elapsed. In orthodox quantum mechanics the standard condition is automatically fulfilled by the imposition of the initial condition $\hat{U}(t_0, t_0) = I$. Furthermore, standarization appears as natural assumptions in more general frameworks (cf. [6, 7]) as a hypothesis assumed "on physical grounds" in the study of the quantum Zeno paradox [6]. In more general frameworks, we think that there exist notably suggested reasons to claim that the standard condition is satisfied. Let the open quantum system formalism be an example [7]. There the system evolution is given by an operator semigroup which satisfies, among other properties, standarization.

The implications of the foregone theorem should be stated clearly. **Assumed** the description of a quantum system by a homogeneous QSES, **if** the system exhibits quantum jumps, **then** the QSES cannot be Markovian. Different attitudes can be adopted. Firstly, the possibility of mathematically representing a quantum jump through an H-valued stochastic process may be neglected, a solution we believe to be too restrictive. Secondly, it can be claimed that the quantum jump does not take place and the evolution of a quantum system, though stochastic, is continuous. This hypothesis is adopted, e.g., in the CSL theory [2]. Nonetheless, it is also possible that the Markov condition not be satisfied even maintaining continuity, as in, e.g. [8]. Finally, the option is left of admitting every hypothesis in the theorem with the subsequent consequences. This alternative has not been studied profoundly yet.

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